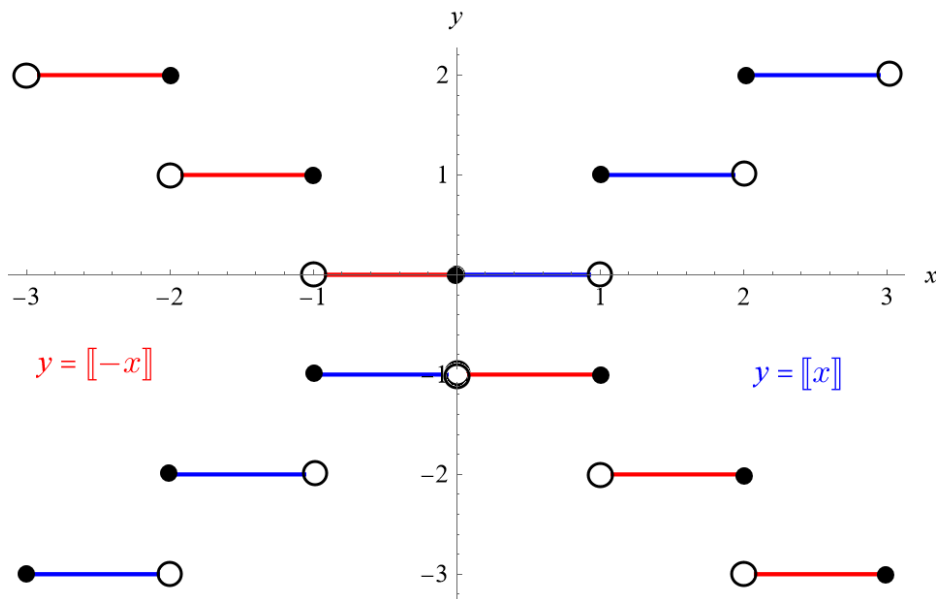


Exercise 55

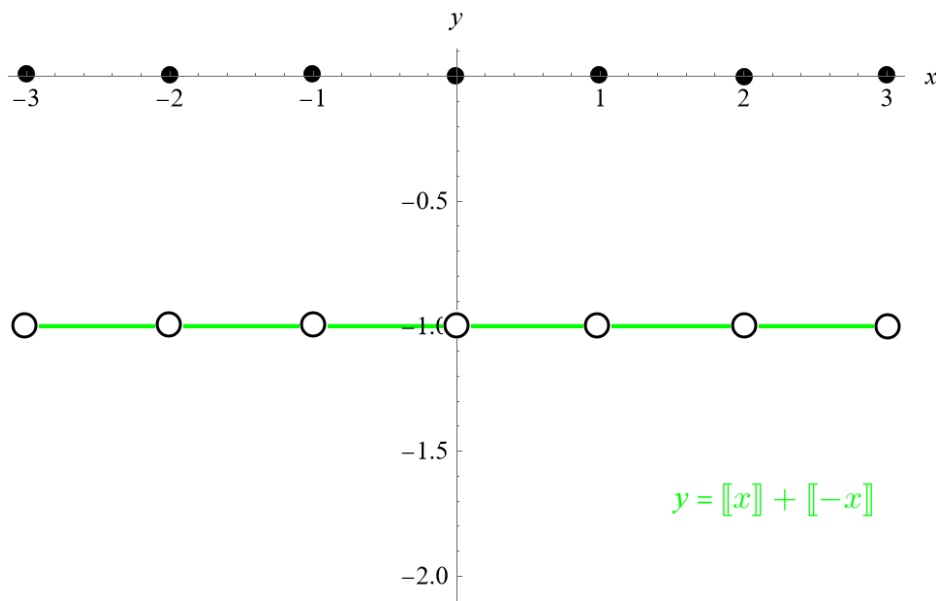
If $f(x) = \llbracket x \rrbracket + \llbracket -x \rrbracket$, show that $\lim_{x \rightarrow 2} f(x)$ exists but is not equal to $f(2)$.

Solution

Below are the graphs of $\llbracket x \rrbracket$ and $\llbracket -x \rrbracket$ superimposed.



With this, the graph of $f(x)$ can be drawn.



Since the left-hand and right-hand limits are both equal,

$$\lim_{x \rightarrow 2^-} f(x) = -1 \quad \text{and} \quad \lim_{x \rightarrow 2^+} f(x) = -1,$$

the limit of $f(x)$ as $x \rightarrow 2$ exists:

$$\lim_{x \rightarrow 2} f(x) = -1.$$

However, it's not equal to $f(2) = 0$.